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On the analyticity in time of solutions of initial boundary value problems for semi-linear parabolic differential equations with monotone nonlinearity

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In this note we consider the following initial boundary value problems:

$$\text{(I.B.V.P.)} \qquad \begin{aligned} &\frac{\partial u}{\partial t} = \Delta u + f(u) \\ &u(t,x)\big|_{\partial\Omega} = 0 \\ &u(0,x) = a(x) \end{aligned} ,$$

where Ω is a bounded domain in R^n with smooth boudary $\partial\Omega$.

The purpose of this note is to report that solutions of (I.B.V.P.) are extensible holomorphically in time t to a sector $\Sigma_\theta = \{ t \in C ; |\arg t| < \theta \}$ in the complex domain which does not depend on initial values, if the nonlinear term $f(u)$ is a monotone decreasing polynomial.

Let us now introduce definitions to state results.

Definition 1. A polynomial with real coefficients $f(u)$ is said to be monotone or to satisfiy condition (M), if $f(0)=0$ and $f'(u) \leqq 0$ for $-\infty < u < +\infty$.

Examples. $f(u) = -u^{2p+1}$, $-u-u^3$, $-u^3-u^5$.

Definition 2. A polynomial with real coefficients $f(u)$ is said to be monotone on $R^+ = [0 , \infty)$ or satisfy condition (M_+), if $f(0) = 0$ and $f'(u) \leqq 0$ for $0 \leqq u < \infty$.

Examples. $f(u) = -u^{2p}$, $-u-u^4$, $-u^2-u^6$.

Theorem 1. Suppose that the nonlinear term $f(u)$ in (I.B.V.P.) satisfies condition (M) and the initial value $a=a(x)$ is real-valued and boundedly continuous in Ω . Then there is a sector $\Sigma_{\theta_0} = \{ t ; |\arg t| < \theta_0 \}$ in the complex domain which is independent of $a(x)$ such that the solution $u(t,x)$ of (I.B.V.P.) is analytically extensible in t to the sector .

Theorem 2. Suppose that the nonlinear term $f(u)$ in (I.B.V.P.) satisfies condition (M_+) and the initial value $a = a(x)$ is a nonnegative and boundedly continuous function. Then there is a sector $\Sigma_{\theta_0} = \{t; |\arg t| < \theta_0\}$ in the complex domain which does not depend on $a(x)$ such that the nonnegative solution $u(t,x)$ of (I.B.V.P.) is analitically extensible in t to the sector.

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